DCC-003-001515 Seat No. _____

B. Sc. (Sem. V) (CBCS) Examination

April / May – 2015

Mathematics : Paper - BSMT-503 (A)

(Discrete Mathematics & Complex Analysis-I)

(New Course)

Faculty Code : 003

Subject Code : 001515

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.

(2) Answer all the MCQs in answer book.

(3) Numbers written to the right indicate full marks of the question.

1 Attempt all the multiple choice questions : 20

(1) If relation R is reflexive, anti-symmetric and transitive then R is said to be _____.

(A) equivalence relation

(B) lattice

(C) POSET

(D) none of these

(2) If R is a equivalence relation on set X then $[x]=[y]\Leftrightarrow$ _____.

(A) $x = y$

(B) xRx

(C) $x \neq y$

(D) xRy

(3) If R is a relation from A to B then

(A) $R \subset A \times B$

(B) $A \times B \subset R$

(C) $R = A \times B$

(D) None of these

(4) (S_{30}, D) is not a _____.

(A) POSET

(B) lattice

(C) totally ordered set

(D) none of these

(5) $a \leq b \Leftrightarrow$ _____.

(A) $a' * b = 0$

(B) $a * b' = 0$

(C) $a' \oplus b = 0$

(D) $a \oplus b' = 0$

(6) If m_i and m_j are distinct minterms in n-variables then _____.

(A) $m_i * m_j = 1$

(B) $m_i \oplus m_j = 1$

(C) $m_i * m_j = 0$

(D) $m_i \oplus m_j = 0$

(7) If x and y are distinct atoms of Boolean algebra $(B, *, \oplus, ', 0, 1)$ then _____.

(A) $x * y = 0$

(B) $x * y = 1$

(C) $x * y = x$

(D) $x * y = y$

(8) Sum of all minterms in n-variables is _____.

(A) n

(B) $n+1$

(C) 0

(D) 1

(9) The Karnaugh map is useful to minimize the _____.

(A) sum of product canonical form

(B) sum of maxterms

(C) product of sum canonical form

(D) product of maxterms

(10) For POSET (S_{30}, D), 5' = _____.

(A) 5

(B) 0

(C) 30

(D) none of these

(11) If $f(z) = e^{2z}$ then imaginary part of $f(z)$ is

(A) $e^x \sin y$

(B) $e^{2x} \sin 2y$

(C) $e^x \cos y$

(D) $e^{2x} \cos 2y$

(12) The imaginary part of $(\sin x + i\cos x)^5$ is

(A) $\sin 5x$

(B) $-\sin 5x$

(C) $\cos 5x$

(D) $-\cos 5x$

(13) If $x + iy = \sqrt{2} + 3i$ then $x^2 + y$ is

(A) 7

(B) 5

(C) 13

(D) $\sqrt{2} + 3$

(14) $f(z) = \overline{z}$ is

(A) analytic

(B) entire

(C) both (A) and (B)

(D) none of these

(15) The real part $\frac{2+3i}{3-4i}$ is

(A) $-\frac{17}{25}$

(B) $\frac{6}{25}$

(C) $-\frac{6}{25}$

(D) none of these

(16) The value of integral $\int_{|z|=2} \frac{\cos z}{z\left(z^2+9\right)} dz$ is

(A) 0

(B) $\frac{i\pi}{2}$

(C) $\frac{i\pi}{4}$

(D) none of these

(17) If C is ellipse with center at origin then $\int_C \frac{\left(z^3+3\right)}{z} dz =$

(A) $6i\pi$

(B) $\frac{i\pi}{2}$

(C) $\frac{i\pi}{4}$

(D) $-6i\pi$

(18) An analytic function with constant modulus is _____.

(A) zero

(B) constant

(C) variable

(D) none of these

(19) If $u = \frac{1}{2}\log\left(x^2 + y^2\right)$ is harmonic then its harmonic conjugate is _____.

(A) $\tan^{-1}\left(\frac{x}{y}\right) + c$

(B) $\tan^{-1}\left(\frac{y}{x}\right) + c$

(C) $\log\left(\frac{x}{y}\right) + c$

(D) $\log\left(\frac{y}{x}\right) + c$

(20) If $C : z = 2e^{i\theta}, 0 \le \theta \le 2\pi$ then $\int_C \frac{e^z}{z+1} dz =$

(A) $\frac{i\pi e^2}{3}$

(B) $-\frac{i\pi e^2}{3}$

(C) 0

(D) none of these

2 (a) Attempt any three : 6

(1) Define :

(i) Relation

(ii) Hasse diagram.

(2) Simplify the Boolean expression

$\left[a * \left(b' \oplus c\right)\right]' * \left[b' \oplus \left(a * c'\right)\right]$.

(3) Draw the Hasse diagram of (S_{60}, D).

(4) In usual notation prove that $A(x) = A - A(x')$.

(5) Prove that (S_{105}, D) is a POSET.

(6) If (P, R) is a POSET then prove that $\left(P, R^{-1}\right)$ is also a POSET.

(b) Attempt any **three** : 9

(1) Define Lattice and show that (S_6, D) is a Lattice.

(2) Express Boolean expression $\alpha(x_1, x_2, x_3) = x_2\, x_3$ in the canonical form

(i) as a sum of product

(ii) as a product of sum.

(3) State and prove cancellation law for distributive lattice.

(4) Prove that the direct product of two lattices is a lattice.

(5) State and prove DeMorgan's law for Boolean algebra.

(6) Show that the lattice with three elements is a POSET.

(c) Attempt any two : **10**

(1) State and prove Stone's representation theorem.

(2) Prove that every chain is a distributive lattice.

(3) Define Boolean algebra and show that $(S_6, *, \oplus, ', 0, 1)$ is a Boolean algebra.

(4) State and prove distributive inequality for lattice.

(5) If $A = \{a, b, c\}$ then show that (S_{30}, D) and $(P(A), \subseteq)$ are isomorphic.

3 (a) Attempt any three : 6

(1) "Every analytic function is entire". True or false ? Justify your answer.

(2) Find real and imaginary part of function $f(z) = \sin z$.

(3) Define analytic function and Harmonic function.

(4) State Cauchy integral formula for derivative.

(5) Show that the function $f(z) = e^{y}(\cos x + i \sin y)$ is not analytic.

(6) State fundamental theorem of algebra.

(b) Attempt any three : 9

(1) Evaluate : $\int_{C} e^{z} dz$; where C is a circle with centre (0, 1) and radius 3.

(2) Prove that $f(z) = \left(z^{2} - 2\right) e^{-x}(\cos x - i \sin y)$ is an analytic function.

(3) Prove that $f(z) = e^{z}$ is an analytic function and hence deduce that $f'(z) = f(z)$.

(4) Show that $f(z) = z^{2}$ is an analytic and entire function. Also find $f'(1)$.

(5) Evaluate : $\int_{C} \frac{dz}{\left(z^{2} + 4\right)^{2}}$; where C is a unit circle.

(6) State and prove Liouville's theorem.

(c) Attempt any two : **10**

(1) State and prove Cauchy's fundamental theorem.

(2) State and prove Morera's theorem.

(3) Evaluate : $\int_{C} \frac{dz}{(z+1)(z-1)}$; where $C : |z| = 2$.

(4) Show that $\sinh x \sin y$ is a harmonic function and find its harmonic conjugate

(5) State and prove Cauchy integral formula.